Computer Labs for Non-Euclidean Geometry

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In collaboration with

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MA 408: Foundations of Euclidean Geometry

What is this? *An examination of Euclidean geometry from a modern perspective. The axiomatic approach with alternative possibilities explored using models.*

Who takes it?

- Math Ed. majors (required for h.s. license)
- Math majors (elective)
- Computer science or engineering majors (elective)

Text-book "Foundations of Geometry" by G. A. Venema.

Euclid’s Elements circa 300 BC.

Postulates =axioms=statements accepted without proof lie in the foundation of a mathematical theory, while all other facts are subsequently derived using logic.

A good system of axioms:

- is consistent;
- is irredundant;
- leads to a non-trivial theory.

Postulates as stated by Euclid circa 300BC
(translated by Sir Thomas Heath)

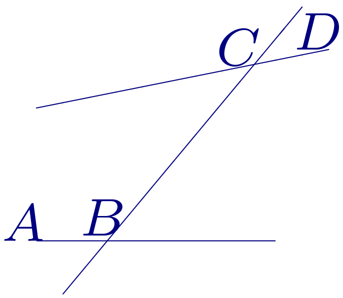
I. To draw a straight line from any point to any point.

II. To produce a finite straight line continuously on a straight line.

III. To describe circle in any center and radius.

IV. That all right angles equal one another.

V. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced infinitely, meet on that side on which are angles are less than the two right angles.



Statements equivalent to the V-th postulate

Definition: Two lines in the plane are parallel if they have no common points.

V. (Playfair, XVIII-th century) For every line l and for every external point P, there exists $\boxed{\text{at most one line}}$ m such that P lies on m and m is parallel to l.

Remarks:

- I.–IV. $\Longrightarrow$ there exists a parallel line, so I.–V. $\Longrightarrow$ there exists the unique parallel line.
- V. $\Longleftrightarrow$ the sum of the measures of angles in any triangle is 180°.
- V. $\Longleftrightarrow$ there exists a rectangle.
- V. $\Longleftrightarrow$ there exists a triangle similar but not congruent to a given triangle.

Attempts to prove the V-th postulate:

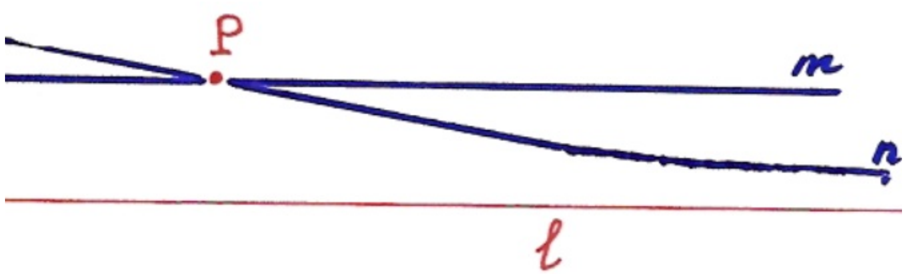
- Omar Khayyám (1050–1123)
- Girolamo Saccheri (1667–1733)
- Johann Heinrich Lambert (1728 –1777),
- Farkas (Wolfgan) Bolyai (1775–1856)

Farkas Bolya to his son János Bolyai:
I have traversed this bottomless night, which extinguished all light and joy of my life. I entreat you, leave the science of parallel alone.

Discovery of hyperbolic geometry.

János Bolyai (in 1820-1823, published 1832) and Nikolai Lobachevsky (published 1829) accepted and studied the geometry in which I.-IV. are satisfied and V. is replaced with:

$\tilde{\mathrm{V}}$. For every line l and for every external point P, there are $\boxed{\text{at least two lines}}$ m and n such that $P \in m$, $P \in n$, $m \,||\, l$ and $n \,||\, l$.



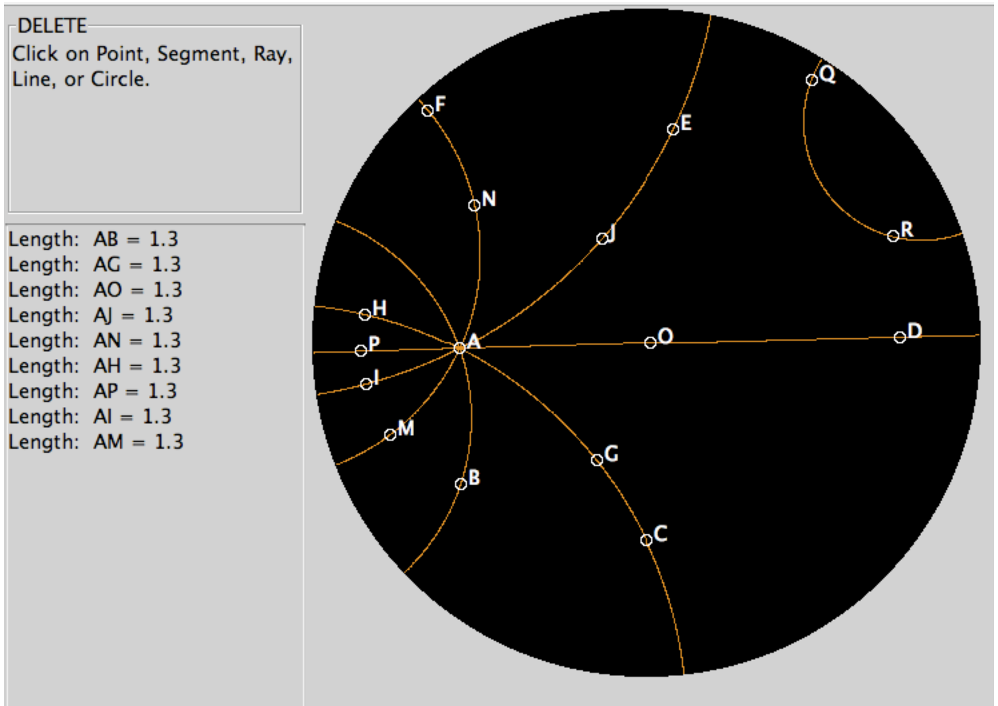
$\tilde{\mathrm{V}}$. $\Longleftrightarrow$ The sum of the measures of angles in any triangle is $< 180^\circ$.

$\tilde{\mathrm{V}}$. $\Longleftrightarrow$ There are no rectangles in hyperbolic geometry.

Poincaré Disk Model of Hyperbolic Geometry

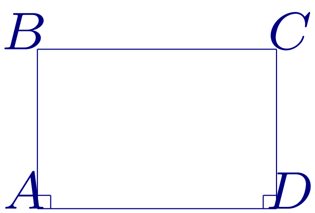
with
NonEuclid Java Software

http://www.cs.unm.edu/∼joel/NonEuclid/NonEuclid.html



Almost rectangle 1.

Sacchery quadrilateral:



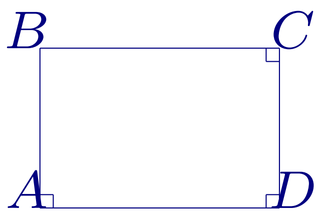
$\Box ABCD$ such that $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$, $\overleftrightarrow{DC} \perp \overleftrightarrow{AD}$ and $\overline{AB} \cong \overline{DC}$

Properties of Sacchery quadrilateral in hyperbolic geometry:

1. $\Box ABCD$ is a parallelogram.
2. The diagonals $\overline{AC}$ and $\overline{DB}$ are congruent.
3. The summit angles $\angle ABC$ and $\angle BCD$ are congruent and acute.
4. $BC > AD$

Almost rectangle 2.

Lambert quadrilateral:



$\square ABCD$ such that $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$, $\overleftrightarrow{DC} \perp \overleftrightarrow{AD}$ and $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$ (three right angles)

Properties of Lambert quadrilateral in hyperbolic geometry:

1. $\square ABCD$ is a parallelogram.

2. $\angle ABC$ is acute.

3. $AB > CD$ and $BC > AD$

Laboratories give students a hands on experience with both Euclidean and non-Euclidean geometry.

- Non-Euclidean
  - Poincaré Model of Hyperbolic Geometry
  - Quadrilaterals in Hyperbolic Geometry
  - Taxicab Metric
- Euclidean
  - Triangle Centers in Euclidean Geometry
  - Circles in Euclidean Geometry
  - Isometries
  - Inversions

Assessment of the influence of technology on student learning

- Teaching the same topic in class during one semester and in laboratory in the other semester.
- Administrating pre- and post- tests.
- Interviews.

The effect of using dynamical software for studying hyperbolic geometry was statistically significant, while for Euclidean geometry was not statistically significant in college classroom.

Publications

1. Hollebrands, K., Smith, R., Iwancio, K., Kogan, I. A. The effects of a dynamic program for geometry on college students understandings of properties of quadrilaterals in the Poincare Disk model. *Proceedings of the 9th International Conference on Mathematics Education in a Global Community* (2007) pp. 613–618.

2. Hollebrands, K., Smith, R., Iwancio, K., Kogan, I. A. College geometry students uses of technology in the process of constructing arguments. *Proceedings of the 29th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education. (T. Lamberg, Ed.)* (2007) 7pp. (electronic)

A quote from student class evaluation:

”The course is fun, and the labs really help to understand more about Euclidean and Non-Euclidean Geometry”