3D MIXED INVARIANT AND ITS APPLICATION ON OBJECT CLASSIFICATION

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ABSTRACT

A new integro-differential invariant for curves in 3D transformed by affine group action is presented in this paper. The derivatives involved are of the first order, and therefore this invariant is significantly less sensitive to noise than classical affine differential invariants, the simplest of which involves derivatives of order 5. A classification procedure based on characteristic curves of an object surface is considered using our proposed mixed invariants. Substantiating examples are provided to verify efficiency and discriminant power of the characteristic spatial curve based 3D object classification.

Index Terms— 3D affine transformation, affine invariant, object classification, invariant feature

1. INTRODUCTION

Curves and surfaces are the fundamental entities in shape/object recognition problems in computer vision and pattern recognition. Their classification under Euclidean, affine, or projective transformations is challenging. A direct comparison of shapes generally requires registration, and the ensuing complexity and difficulty in its application in many important problems have recently led to a renewed research interest in "transformation invariant".

Differential invariants, such as Euclidean curvature and torsion for space curves, are the most classical. The affine and projective counterparts of curvature and torsion may also be defined. The practical utilization of differential invariants is, however, limited due to their high sensitivity to noise.

This motivated the high interest in other types of invariants such as semi-differential, or joint invariants [2, 7, 6] and various types of integral invariants [3, 10, 8]. Lin and Hu[11] extended the continuous integral to a discrete setting and proposed a "Summation Invariant".

Integral invariants have advantage in applications, because integration smoothes noise out and hence induces numerical robustness. In particular these invariants were successfully applied to face recognition [12].

Explicit expressions for integral invariants, however, appear to only be known for plane curves in 2D, and have thus far remained elusive for spatial curves in 3D, primarily due to their complexity. With an increasing availability of 3D data acquisition systems and subsequent emerging applications, the interest has shifted to 3D integral invariants. To achieve a trade-off between computational complexity of computation and numerical robustness, we derive a *mixed integro-differential invariant*, that depends on the first order derivatives and integral variables.

In Section 2 we derive a novel mixed invariant, that depends on the first order derivatives and integral variables. The integral variables are 3D analogs of the potentials introduced in [8] for plane curves. In Section 3, we discuss an application of this invariants to extracted curve features from 3D objects for a subsequent classifications applications. We provide some concluding remarks in Section 4.

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2. MIXED INVARIANT

Fels and Olver [4][5] generalized Cartan's[1] method for computing differential invariants, so that it become applicable for computing various types of invariants. Hann *et al* [8] introduced integral variables and used Fels and Olver construction to derive integral invariants for curves in 2D. Lin *et al* [11] implemented the algorithm by turning integrals into summations. In this section, we use Fels-Olver construction and the 3D analog of Hann-Hickerman integral variables, to derive a mixed integro-differential invariant for curves in 3D transformed by the affine group.

2.1. 3D Affine Transformation

The full affine group action on $\mathbb{R}^3$ may be written as:

$$(x, y, z) \mapsto (a_{11}x + a_{12}y + a_{13}z + a_{14}, a_{21}x + a_{22}y + a_{23}z + a_{24}, a_{31}x + a_{32}y + a_{33}z + a_{34})$$

where $a_{ij} \in \mathbb{R}$ for $i \in 1, 2, 3, 4, j \in 1, 2, 3, 4$ and

$$det \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \neq 0.$$

This group action may also be represented by the matrix

$$G = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Our construction of invariants in the sequel will call on pointwise definite integrals which will in turn require an initial point on a curve, which we denote by (x_0, y_0, z_0). This will, as a result, eliminate translation parameters by merely readjusting every other point relatively to (x_0, y_0, z_0). The simplified transformation may thus be written as:

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \quad (1)$$

This simplification will help reduce the total number of parameters and hence of the number of equations as discussed next. Note that the invariant we obtain for this simplified group can always be converted back to that of the full affine group, by replacing (x, y, z) by $(x - x_0, y - y_0, z - z_0)$.

2.2. Extending Group Actions

Differential invariants for curves in $\mathbb{R}^n$ are obtained by prolonging the group action on $\mathbb{R}^n$ to the jet space J^k, parametrized by coordinates of the curve and their derivatives up to the order k. To obtain an invariant the total number of variables should exceed the dimension of the group.

Han *et al* [8] prolonged the action to integral variables, called, potentials, and derived the integral invariants for curves in 2D. A similar approach may be adopted for a 3D setting. To that end, we define potentials $D^{i,j,k}$, $H^{i,j,k}$ and $L^{i,j,k}$ of order l as:

$$D^{i,j,k} = \int_0^x x^i y^j z^k dx, j + k \neq 0$$

$$H^{i,j,k} = \int_0^y x^i y^j z^k dy, i + k \neq 0$$

$$L^{i,j,k} = \int_0^z x^i y^j z^k dz, i + j \neq 0$$

where $i + j + k = l$.

By factoring out the translation, we reduced the action linear group of dimension 9. The potentials up to second order are sufficient to obtain an invariant. We define the following integral variables,

$$r = D^{0,1,0}, s = D^{0,1,0}, t = H^{0,0,1}, u = D^{1,1,0}$$

$$v = D^{1,0,1}, w = D^{0,1,1}, m = H^{1,0,1}, n = H^{0,1,1}$$

$$o = L^{0,1,1}, p = D^{0,2,0}, q = D^{0,0,2}$$

The affine action is prolonged to these variables, and Fels-Olver method is applied for finding invariants in $\mathbb{R}^{14}$ parametrized by

$$(x, y, z, r, s, t, u, v, w, m, n, o, p, q).$$

Solving the resulting system of equations for all group parameters in this space, as required by Fels-Olver method, quickly becomes intractable. As a tradeoff between the computational complexity of solving the system of equations to obtain an integral invariant, and the numerical sensitivity of differential invariants to noise, we obtain a hybrid invariant which utilizes first order (least sensitive to noise) derivatives and integral auxiliary variables.

To this end we prolong the action to $\mathbb{R}^{16}$ parametrized by

$$(x, y, z, y', z', r, s, t, u, v, w, m, n, o, p, q),$$

where

$$y' = \frac{dy}{dx}, z' = \frac{dz}{dx}.$$

We use explicit formulas for the transformations of variables

$$(x, y, z, y', z', r, s, t, u, q) \rightarrow (\bar{x}, \bar{y}, \bar{z}, \bar{r}, \bar{s}, \bar{t}, \bar{y}', \bar{z}', \bar{u}, \bar{q}),$$

which are omitted due to space limitations, except for $\bar{x}, \bar{y}, \bar{z}$ given by (1) above and $\bar{q}$ given by (2) below.

2.3. Affine Invariant in 3D Space

Following Fels-Olver procedure we choose a valid cross-section

$$(x, y, z, r, s, t, y', z', u) = (0, 0, 1, 1, 1, 1, 1, 1, 0).$$

We then solve equations

$$(\bar{x}, \bar{y}, \bar{z}, \bar{r}, \bar{s}, \bar{t}, \bar{y}', \bar{z}', \bar{u}) = (0, 0, 1, 1, 1, 1, 1, 1, 0)$$

to find the group parameters parameters

$$(a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33})$$

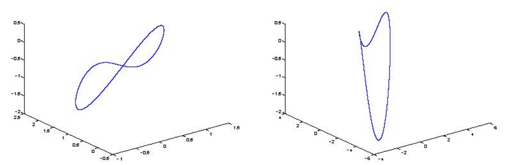
that bring an arbitrary point to the cross-section.

The solution is shown in Appendix A. The mixed integro-differential invariant is obtained by substitution of those expressions into the remaining non-normalized variable:

$$\begin{aligned} \bar{q} &= a_{11}(1/3\, a_{31}{}^2 x^3 + a_{32}{}^2 p + a_{33}{}^2 q + 2\, a_{31} a_{32} u \\ &+ 2\, a_{31} a_{33} v + 2\, a_{32} a_{33} w) + a_{12}(a_{31}{}^2 \left(x^2 y - 2\, u\right) \\ &+ 1/3\, a_{32}{}^2 y^3 + a_{33}{}^2 \left(y z^2 - 2\, o\right) + a_{31} a_{32} \left(x y^2 - p\right) \\ &+ 2\, a_{31} a_{33} m + 2\, a_{32} a_{33} n) + a_{13}(a_{31}{}^2 \left(x^2 z - 2\, v\right) \\ &+ a_{32}{}^2 \left(y^2 z - 2\, n\right) + 2\, a_{31} a_{32} \left(x y z - m - w\right) \\ &+ a_{31} a_{33} \left(x z^2 - q\right) + 1/3\, a_{33}{}^2 z^3 + 2\, a_{32} a_{33} o) \end{aligned}$$

2.4. An Example

Consider two 3D spatial curves Fig. 1-a and Fig. 1-b for comparison. Fig. 1-b is related to Fig. 1-a by a full affine transformation. Since the invariant is a ratio its value blows up at the points on the curve where the denominator is zero. We selectively remove all numerical instability around these points.The result is shown in Fig. 2. No obvious deviation between the two invariants can be detected.



(a) (b)

Fig. 1. *(a)3D curve 1(b)3D curve 2*

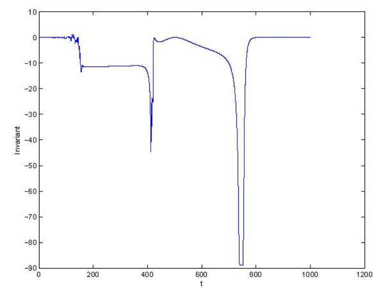


Fig. 2. *the mixed invariant for curves 1 and 2*

3. 3D OBJECTS CLASSIFICATION

A problem of curve comparison under affine transformations arise in many applications, in particular in biometrics [12] and 3D object clustering. In the present example, we consider application to classification of 3D objects based on a set of characteristic spatial curves.

3.1. Experimental Design

The Princeton Shape Benchmark[13] provides a repository of 3D models. A subset of four models are shown in Fig. 3.1. We may assume that the characteristic curves have already been extracted from 3D models in Princeton Shape Benchmark, as shown in Fig. 4. There are totally 50 characteristic curves, and each of them are re-sampled to 5000 points. Apply 10 randomly generated 3D affine transformations to these curves, and 10 variations for each curve are generated (Fig. 5)[1]. The problem is to classify all of these curves. To make this problem even more challenging and to illustrate the noise sensitivity, gaussian noise with distribution $N(0, \sigma^2)$ is added to each of the variation.

The discrimination power and sensitivity to noise are analyzed using the error rate of classification of the proposed mixed invariant and classical differential invariants, called affine curvature, are compared. (See [9] for the expression of the affine curvature in terms of the Euclidean invariants). Two sets are required for classification purpose, namely training set and classification set. The training set is obtained by randomly selecting three variations out of ten from each characteristic curve. The 7 left for each characteristic curve automatically form the testing set. Such a classifier is implemented as a Nearest Neighbor (NN) Classifier in Euclidean Space using a L^2 distance as a metric.

[1]These curves would undergo such transformations when the 3D object is subjected to a transformation.

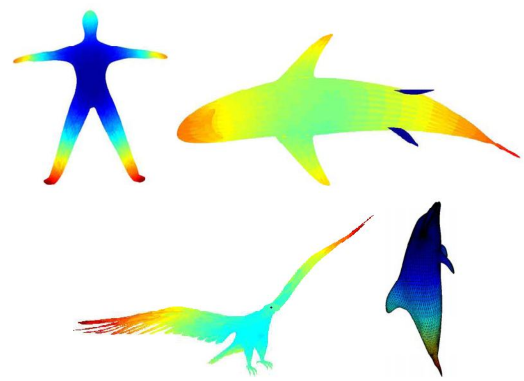


Fig. 3. 3D models from The Princeton Shape Benchmark(Best visualized in color)

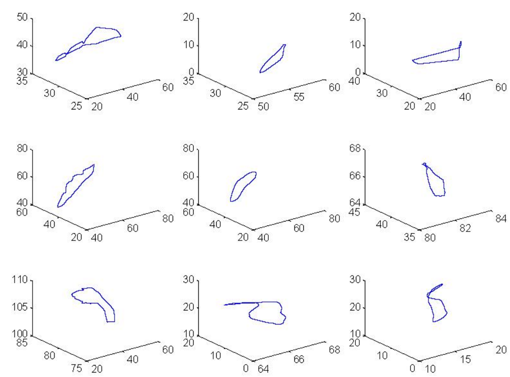


Fig. 4. *3D spatial feature curves*

3.2. Experimental Results

Two experiments are carried out with different noise variance, namely $\sigma = 0.1$ and $\sigma = 1$. The error rates of the two sigma settings are shown in Table.I.

Due to higher order derivatives in differential invariants, the error rates are over 60%, which makes the differential invariants practically useless. With only the first order derivatives and integrals, the Mixed Invariant reduces the error rate dramatically from 60% to 10% and provides a practical solution to classify curves under affine transformation. But the

Table 1. error rate

	Mixed Invariant	Differential Invariant
$\sigma = 0.1$	0.0971	0.6086
$\sigma = 1$	0.1829	0.7314

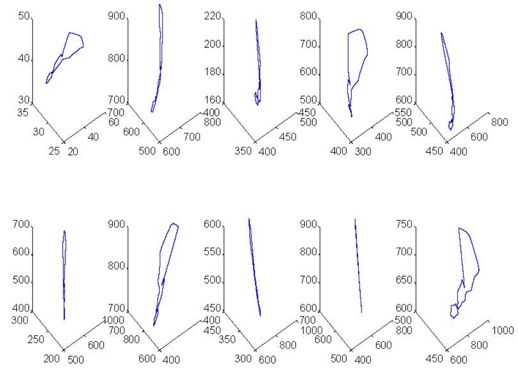


Fig. 5. *10 variations of a curve under affine transformation*

presence of the first order derivatives in the Mixed Invariant, results in a modest increase of error rate as the noise variance is increased by an order of magnitude.

4. CONCLUSIONS

In this paper, we presented a new mixed invariant for curves in 3D with respect to affine transformation. This invariant depends on the first order derivatives and integral variables. Our future work will focus on obtaining invariants which are fully integral which will in turn provide additional robustness. An application to classification of characteristic curves of a 3D object as they are subjected to random affine transformation is considered.

5. ACKNOWLEDGEMENT

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7. APPENDIX A

$$
\begin{aligned}
a_{11} &= (6\,znx - 6\,zwy + 6\,yox + 3\,z^2p + 3\,y^2q \\
&\quad -4\,z^2xy^2)n_4/((n_1+n_2)n_3) \\
a_{12} &= (3\,xqy + 6\,uz^2 + 6\,xmz - 6\,vyz - 5\,x^2yz^2 \\
&\quad +6\,x^2o)n_4/((n_1+n_2)n_3) \\
a_{13} &= (x^2y^2z - 6\,uyz + 6\,nx^2 + 3\,xpz + 6\,vy^2 \\
&\quad -6\,wxy - 6\,mxy)n_4/((n_1+n_2)n_3) \\
a_{31} &= (y^2z^2 - 4\,y^2z'\,s - y^2xz'\,z + 2\,yzrz' + 2\,yxz'\,t \\
&\quad -4\,yzt + yz^2y'\,x + 6\,y'\,syz + 4\,rz'\,t + 4\,t^2 \\
&\quad -2\,ty'\,zx - 4\,y'\,st - 4\,y'\,z^2r)/(n_3n_4) \\
a_{32} &= (x^2z'\,yz + 6\,sz'\,yx - 2\,syz - 3\,yz^2x - 4\,tx^2z' \\
&\quad -4\,z'\,sr - 2\,rxz'\,z + 6\,tzx + 4\,rz^2 - 4\,xsy'\,z \\
&\quad -4\,st + 4\,s^2y' + y'\,z^2x^2)/(n_3n_4) \\
a_{33} &= (x^2z'\,y^2 - 3\,x^2y'\,yz + 4\,x^2ty' - 4\,xyz'\,r \\
&\quad +3\,xy^2z + 6\,xry'\,z - 6\,xyt - 2\,xysy' + 4\,r^2z' \\
&\quad -6\,rzy + 4\,sy^2 + 4\,rt - 4\,y'\,sr)/(n_3n_4)
\end{aligned}
$$

$$
\begin{aligned}
n_1 &= -3\,xz'\,zp + 3\,z^2p - x^2z'\,zy^2 + 6\,znx \\
&\quad + 6\,y'\,zz' - 6\,z'\,vy^2 + 3\,y^2q - 6\,x^2z'\,n \\
&\quad + 6\,xz'\,my + 6\,xz'\,wy \\
n_2 &= 5\,y'\,z^2x^2y - 3\,y'\,xqy - 6\,zwy - 6\,y'\,z^2u - 6\,yox \\
&\quad - 6\,y'\,zxm + 6\,z'\,zuy - 6\,y'\,x^2o - 4\,z^2xy^2 \\
n_3 &= xyz - 2\,tx + 2\,sy - 2\,rz \\
n_4 &= 2\,y'\,s + yxz' + yz - 2\,rz' - 2\,t - y'\,zx
\end{aligned}
$$