

# 3D FACE RECOGNITION USING EUCLIDEAN INTEGRAL INVARIANTS SIGNATURE

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## ABSTRACT

A novel 3D face representation and recognition approach is presented in this paper. We represent a 3D face by a set of level curves of geodesic function starting from the nose tip, which is invariant under isometric transformation of the surfaces. A pose change induces a special Euclidean transformation (a composition of a rotation and a translation) of the surface that represents a face and leads to the Euclidean transformation of the iso-geodesic curves. A change of facial expression induces isometric transformation of the iso-geodesic curves. Although the set of isometric transformations of a surface is larger than the set of Euclidean transformations in 3D, we assume that iso-geodesic curves undergo piecewise Euclidean transformations, i.e. the transformation of relatively small segments of the level curves is Euclidean. A Euclidean invariant integral signature for curves in 3D is presented in this paper. Euclidean invariant integral signature provides a classification of spatial curves which is independent of their position in 3D space and parameterization, and is not sensitive to noise. A recognition procedure based on comparing face feature in the invariant signature space is proposed. Substantiating examples are provided with an achieved classification accuracy of 95% for faces with various poses and facial expressions.

## 1. INTRODUCTION

Face recognition has been extensively studied for over 30 years, and its complexity together with its relevance in security and surveillance problems have recently led to a renewed research interest. Techniques in 2D face recognition abound, and Zhao and Chellappa [1] provide a fairly comprehensive account of the state of the art. Variation of lighting and pose in 2D face recognition are widely recognized to be major impediments to the deployment of robustly performing algorithms. Specifically, the performance of many existing algorithms greatly deteriorates when the training and testing sets do not share a significant number of common views and lighting conditions. The recently developed 3D scanning techniques are believed to provide a potential to alleviate the limitation due to lighting and pose, and their rapid deployment would go far in paving the way for a viable recognition system. Exploiting such data amounts to extracting the intrinsic geometric information and utilizing it as the basis for characterizing individual faces.

Recent research activity in 3D and multi-modal face recognition techniques is reviewed in Chang *et al.*[2]. We can distinguish two main classes of data driven techniques:

- A 3D (geometric-only) approach includes a curve and profile-based description of a face by Nagamine *et al.*[5] and Baumier [9], a volumetric approximative representation of a face proposed by Irganoglu *et al.* [8], a Iterative Close Point based algorithm by Cook *et al.* [7], a planer curve based method by Samir *et al* [14], and a polar geodesic representation based approach by Mpiperi *et al* [13].
- A multi-model (geometric and photometric) approach includes several PCA based methods. Chang *et al.*[3] implement PCA to both 3D and 2D images, Tsalakanidou *et al.*[10] extract depth and color eigenfaces, and Bronstein, Bronstein and Kimmel [6] use eigendecomposition of flattened textures and Multidimensional scaling (MDS)-based canonical images. Also related is 3D Point Signatures and 2D Garbor Filter responses based technique proposed by Wang [11]. Lu and Jain [12] jointly exploit range and texture information with a Linear Discriminant Analysis to construct a hierarchical system.

In this paper, we represent a face by a surface in 3D, and propose in Section 2 a novel technique which bases an accurate description of a face range image on a set of curves intrinsic to the surface. To better contend with face pose and expression variability, we propose in Section 3 the Euclidean an Integral Invariant Signature. The scale independent Euclidean integral invariant signature that which provides a noise tolerant classification method for curves up to rotation, translation and scale. Section 4 discusses the experiment that verifies the robustness of the proposed classification approach under affects of noise and change of facial expressions. The achieved classification rate of 95 % which is promising. We provide the conclusion in Section 5.

## 2. FACE REPRESENTATION

General 3D object representation has been extensively studied, and various approaches[16] have been proposed in the literature. When faces are subject to transformations, especially Isometric transformation(under effect of pose and facial expression), most representations will vary for the same subject. It is well known that a Geodesic distance between points on the surface is invariant to isometric transformation. This property has been exploited in several approaches [17] [23] [21][19],[26] and [6] , [14], [13] extend this idea to face recognition since it is invariant to facial expression as verified in [6], [13]. We adopt here the similar idea to guide the curve extraction, and construct curve based 3D face representation.

## 2.1. Geodesic Distance Function

Geodesic distance between two points on a manifold is the shortest path between these two points along the manifold. Although the Euclidean distance between two points may change under different facial expressions, the geodesic distance, only changes very slightly [6], [13], and the changes may be ignored. We, hence, may pre-define the nose tip as a reference point, and the Geodesic Distance Function (GDF) at any point on the 3D face is defined as the geodesic distance between this point and the nose tip.

A practical problem is the movement of the mouth. Opening mouth may generate a hole, which changes the topology of the face. The geodesic distance from the nose tip to the area under the mouth may change as a result of the opening mouth. One possible solution is suggested in [6] to remove the mouth region, and we always assume there is a hole. Here we adopt this idea. A mouth is easy to locate with texture information, and the test data we are using provides both the geometrical and texture information. Using level set approach, the mouth region may be located and removed from the face. We omit the details of this technical approach for it is not essential to our work and for space reasons.

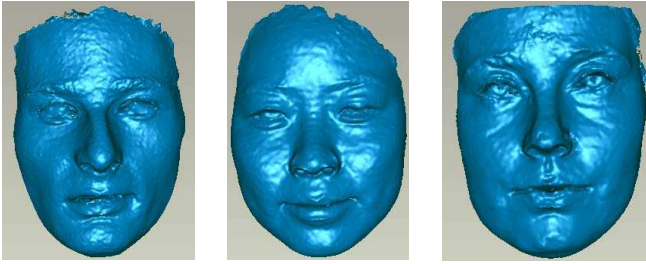


Fig. 1. 3D faces

The GDF of several faces in Fig. 1 are shown in Fig. 2 (best viewed in color). The color of the object in Fig. 2 indicates the GDF value at each point on the surfaces of the object.

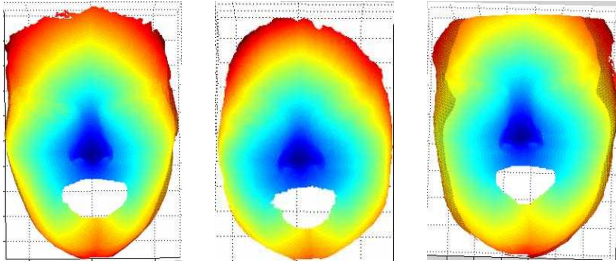


Fig. 2. Geodesic Distance Function of 3D faces

## 2.2. Iso-Geodesic Curves

GDF (denoted as  $g$ ) is a continuous function on the surface of an object. Within the surface, iso-geodesic curves of level  $c$  are defined as curves satisfying the following condition:

$$g(x, y, z) = c.$$

Let  $n$  be the total number of iso-geodesic curves extracted from a face. In our current approach, we set maximum level  $c_n$  to be

the curve passing the outer corner of the eyes, so that we are only focused on the  $n$  curves from the nose tips to the outer corner of the eyes, which covers most of the region of interest in 3D faces.

For a triangulated meshed surface, the vertex with the exact level  $c$  may not exist. However, under a reasonable assumption that the GDF is a linear function along each edge, the vertex with exact level  $c$  may be interpolated linearly. Specifically, it follows two steps:

1) Locate the edge

For any edge starting from  $v_s$  and ending at  $v_e$ , calculate:

$$S = (g_n(v_s) - c)(g_n(v_e) - c)$$

Under the linearly increasing assumption of a GDF, it is easy to see that  $c_i$  is located on the edge where  $S \leq 0$ .

2) Locate the vertex by linear interpolations

Once an edge is selected in step 1, the exact location of a vertex may be linearly determined as:

$$v_c = \frac{c - g_n(v_s)}{g_n(v_e) - g_n(v_s)}(v_e - v_s) + v_s$$

The iso-geodesic curves at level  $c$  may be generated by connecting the vertices on the same face and nearby faces sharing the same vertex. Several examples of iso-geodesic curves of surfaces are shown in Fig. 3 as space curves in 3D. Each color indicates one level  $c_i = c_n i/n$  for a total of 20 levels (we chose  $n = 20$ ) and  $i = 1, 2, \dots, n$  in our current experiment.

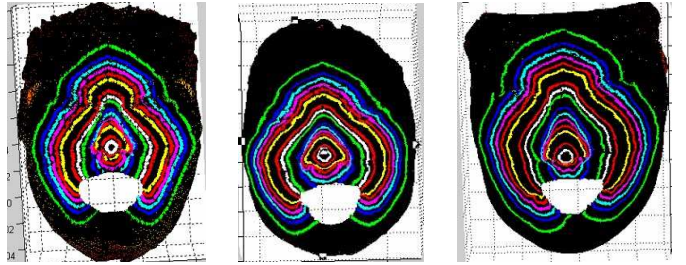


Fig. 3. Iso-Geodesic curves of 3D faces

## 3. INVARIANT FEATURES

The Iso-geodesic curves capture the same sets of points on a face under different pose and facial expression. As such, it is a robust representation of 3D faces, and the face recognition may be done by comparing the space iso-geodesic curves. Space curves undergo transformations, which makes the comparison difficult. A direct comparison of curves, such as shape matching, generally requires registration, which is complicated and difficult in its application in many important problems. Geometric Invariants have as a result been of great research interest.

The classical Euclidean Differential invariants are the Curvature and Torsion, whose practical utilization is limited due to their high sensitivity to noise. To smooth noise out, a variety of integral invariants [24, 22, 20] are proposed. Among them, [22] developed a robust integral invariant signature in 2D, which is independent of parametrization and initialization of curves. Inspired by [22], we developed Euclidean Integral Invariant Signature in 3D.

### 3.1. 3D Integral Invariant Signature

For a closed/open space curve in 3D, the scaling effect may be cancelled by normalizing the total arc-length to be 1. Similarly to [22], we also focus on local regions to make the signature independent of the starting point of a curve. The local regions are obtained by dividing a curve into small segments of equal arch length. To construct a signature, two invariants are required for each segment. However, the area criterion in [22] is not easily defined for space curves in 3D, requiring us to adopt a different strategy, namely the Moving Frame approach [15]. We use Fels-Olver [15] construction and the 3D analog of Hann-Hickerman integral variables[20], to derive integral invariants for curves in 3D subjected to the Euclidean group.

### 3.2. 3D Euclidean Transformation

The rotation group action on  $\mathbb{R}^3$  may be represented by the matrix:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & -\sin(\psi) \\ 0 & \sin(\psi) & \cos(\psi) \end{bmatrix} \times \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \\ \times \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

yielding a 3D Euclidean Transformation from  $(x, y, z)$  to  $(x_e, y_e, z_e)$  as

$$\begin{pmatrix} x_e \\ y_e \\ z_e \end{pmatrix} = R \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

Our construction of invariants depends on line integrals. An initial point on a curve denote by  $(x_0, y_0, z_0)$ , is necessary. This will, as a result, eliminate translation parameters by merely readjusting every other point relatively to  $(x_0, y_0, z_0)$ . Automatic selection of an initial point for a closed curve may required additional processing. Fortunately, we are dealing with open segments, whose end point can easily serve as the initial point. And the Euclidean transformation is hence reduced to a rotation:

$$\begin{pmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{pmatrix} = R \times \begin{pmatrix} X \\ Y \\ Z \end{pmatrix},$$

where  $X = x - x_0, Y = y - y_0, Z = z - z_0$ .

### 3.3. Extending Group Actions

To detive the invariants, we first prolong the action of the group to integral variables, called, potentials. Similar potentials were introduced by Han *et al* [20] in 2D, which we extend into 3D by defining:  $X_{ijk}, Y_{ijk}$  and  $Z_{ijk}$  of order  $l$  as:

$$X_{ijk} = \int_0^t X^i(t)Y^j(t)Z^k(t)dX(t), j + k \neq 0$$

$$Y_{ijk} = \int_0^t X^i(t)Y^j(t)Z^k(t)dY(t), i + k \neq 0$$

$$Z_{ijk} = \int_0^t X^i(t)Y^j(t)Z^k(t)dZ(t), i + j \neq 0$$

where  $i + j + k = l$  and  $t$  is the parameter for a curve, such as arch length.

By factoring out the translation, we reduce the action to the group of rotation with dimension 3. We hence use the following integral variables,

$$Z_{010}, Z_{100}, Y_{100}, Z_{011}.$$

The Euclidean action is prolonged to these variables, and Fels-Olver method is applied in order to find invariants in  $R^6$

$$(X, Y, Z, Z_{010}, Z_{100}, Y_{100}, Z_{011}).$$

### 3.4. Euclidean Invariant in 3D

Following Fels-Olver procedure we choose a valid cross-section

$$(Y, Z, Z_{011}) = (0, 0, 0).$$

We next solve the transformed equations

$$(\bar{Y}, \bar{Z}, \bar{Z}_{011}) = (0, 0, 0)$$

to find the three group parameters parameters

$$(\theta, \phi, \psi)$$

that bring an arbitrary point to the cross-section.

The Euclidean Integral Invariants are obtained by substitution of

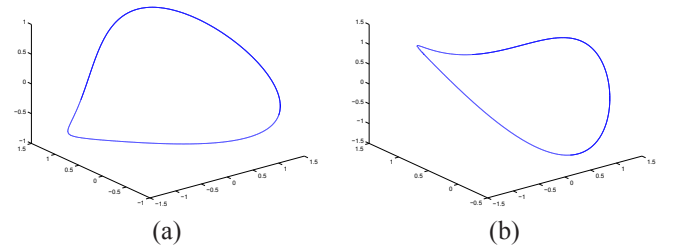
$$i_1 = \bar{X} = \sqrt{X^2 + Y^2 + Z^2} \\ i_2 = \bar{Z}_{010} = \frac{(XYZ - 2XZ_{010} + 2YZ_{100} - 2ZY_{100})^2}{4\sqrt{X^2 + Y^2 + Z^2}}$$

Simplifying  $i_1$  and  $i_2$  yield two Euclidean Invariants  $I_1$  and  $I_2$ :

$$I_1 = i_1 = \sqrt{X^2 + Y^2 + Z^2} \\ I_2 = \sqrt{4i_1i_2} = XYZ - 2XZ_{010} + 2YZ_{100} - 2ZY_{100}$$

Since one end of the segment coincides with the origin,  $I_1$  is the Euclidean distance between two end points. Instead of area in [22],  $I_2$  is some volume defined for the segment.

The integral signature of a space curve in 3D is the variation of one independent invariant  $I_1$ , evaluated on the curve, relative to another  $I_2$ .



**Fig. 4.** (a) original curve (b) transformed curve

The signature of two versions (in Fig. 4) of a curve under Euclidean transformation is shown in Fig. 5.

For curves under isometric transformation, such as iso-geodesic facial curves, we may assume that most segments are locally under

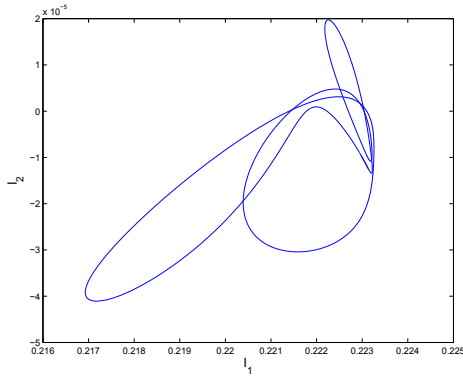


Fig. 5. Integral Invariant Signature

Euclidean transformation locally. In this case, we can still use the integral invariant signature to characterize surfaces. The Euclidean transformed segment will display a similar signature. The only difference will be in the isometric transformed segment. For example, the signatures of the two curves in Fig. 6 are shown in Fig. 7. We may notice that most parts of the signature are overlapped. The only difference near the star correspond to the articulate regions in Fig. 6. Any similarity measurement may indicate that these two signatures are highly similar.

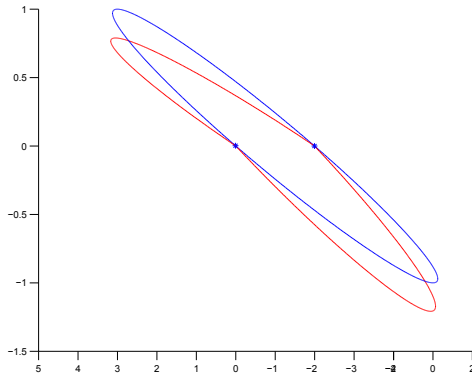


Fig. 6. Curves undergoing isometric transformation

### 3.5. Curve Matching

With the integral invariant signature, the space curves in 3D are mapped to the 2D invariant space, where there is no transformation effects, and matching space curves under transformation in 3D is reduced to matching signatures. A signature curve is parameterized by arch-length starting at the  $\min(I_1)$  point, and the signature may be represented by a vector. The cosine similarity is used to measure the similarity between signatures. And the similarity between objects is the sum of similarities between sets of curves.

## 4. EXPERIMENTAL RESULT

The experimental data we are using in this paper is FRGC2[25]. The data from spring 2003 are the training set, which contains 222 sub-

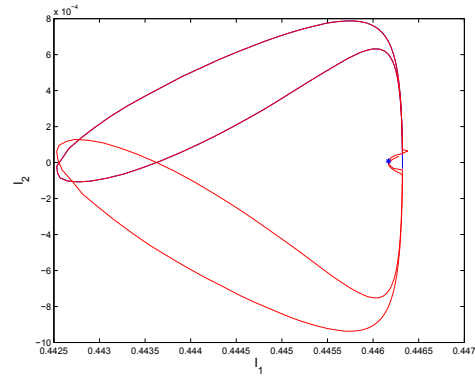


Fig. 7. Signature for Curves undergoing isometric transformation

jects, and the number of scans of each subject varies 1 to 10. The data from the same 222 subjects are selected from spring 2004, serving as testing data. The number of 3d scans per subject varies from 1 to 12.

Following the procedures in the above two sections, each face is represented by 20 curves and the signature of each curve is constructed and sampled to a 200 dimensional vector. The cosine similarity between the corresponding signature is calculated as the similarity measurement. In this experiment, we use One Nearest Neighbor classification rule, and the accuracy is 95%, which is numerically comparable with most recently proposed geometry driven techniques in FRVT 2006 Large-Scale Results. By using the integral invariant signature, we successfully avoid the time consuming registration procedure, which makes it a fast real time system in theory.

## 5. CONCLUSIONS

In this paper, we presented a new geometric invariant-based method for 3D face recognition. Each face may be represented by 20 iso-geodesic curves, which gives us a systematic way to capture the same sets of points under iso-metric transformations. The pose and facial expression effect may be eliminated by mapping the space iso-geodesic curves to Euclidean integral invariant signature space. Substantiating examples are provided with an achieved classification accuracy of 95%.

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