COLLEGE GEOMETRY STUDENTS' USES OF TECHNOLOGY IN THE PROCESS OF CONSTRUCTING ARGUMENTS

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Prior research on students' uses of technology in the context of Euclidean geometry has suggested it can be used to support students' development of formal justifications and proofs. This study examined the ways in which students used a dynamic geometry tool, NonEuclid, as they constructed arguments about geometrical objects and relationships in hyperbolic geometry. Five students enrolled in a college geometry course participated in an interview about properties of quadrilaterals in the Poincaré disk model. Toulmin's argumentation model and the MATCI framework were used to analyze students' uses of technology in the process of constructing arguments as they were working on various tasks.

Introduction and Theoretical Framework

Traditionally, College Geometry is a difficult course for students because it requires them to reason strictly from axioms and postulates rather than informal experiences and intuitive understandings. In order for students to appreciate the importance of the rigorous axiomatic approach, most college geometry courses introduce students to a less intuitive world of non-Euclidean Geometry. Students generally enter a college geometry course with twelve or more years of experience working within the Euclidean system of axioms, and their understandings of figures and relationships within this system are challenged when the axioms are modified. While geometry, in general, is a very visual subject, there are several limitations to students' uses of paper-and-pencil diagrams, especially when it comes to non-Euclidean geometries. A student may create inaccurate misleading diagrams and arrive to incorrect conjectures, or a student may create a correct diagram that is too specific and this may inhibit their ability to derive general conclusions and proofs that go beyond the drawing they have created (Schoenfeld, 1986).

Many mathematics education researchers and professional organizations have suggested the use of dynamic software programs to teach geometry (NCTM, 2000). These software programs enable students to construct accurate diagrams and interact with the diagrams to abstract general properties and relationships, because the ways in which the programs respond to the students' actions is determined by geometrical theorems. Research related to secondary students' uses of such programs has been shown to improve their understandings of geometrical concepts and support their development of formal proofs (Laborde, Kynigos, Hollebrands & Straesser, 2006). Such programs show promise for working with models of hyperbolic geometry, where interpretations of planes, lines, and angles are unconventional. Various technological tools have been developed to assist students in reasoning within different non-Euclidean systems (e.g., *NonEuclid*, Castellanos, 2007), but little research has examined how students' uses of such tools affects their mathematical thinking or influences the mathematical arguments they develop.

While proof and formal logic are two characteristics of mathematics that separates it from other sciences, students learning mathematics often engage in mathematical reasoning and sense-making activities prior to constructing a formal proof. It is in this "territory before proof" (Edwards, 1997) where students make the most use of technology tools, but few researchers have examined how students' use of such tools affects less formal mathematical arguments they develop. The purpose of this study is to investigate the ways in which students use dynamic geometry tools (*NonEuclid*) as they construct arguments about geometrical objects and relationships in hyperbolic geometry. The purposes of this study are closely tied to the goals of PME-NA in that it seeks to build upon and extend our knowledge of the psychological processes involved in students' construction of mathematical arguments when solving mathematical tasks with the use of a technology tool.

One model that has been used by several researchers (e.g., Stephan & Rasmussen, 2002; Lavy, 2006) in mathematics education to examine students' mathematical arguments is Toulmin's model of argumentation (Toulmin, 1958). This model views argumentation from a practical perspective rather than a pure logico-mathematical viewpoint. This model decomposes an argument into three main components: claim, data, and warrant. When making an argument a claim is made. The claim is often the purpose of the argument and it is generally based on some form of evidence, facts or general information called data. If the argument is challenged, then a warrant may be provided, which is the logical connection between the data and the claim. The warrant explains the relationship between the claim and the given data. To aide the audience in understanding the reasoning used in the warrant, additional information, called backing, may be provided to support the warrant. This model of argumentation was used to focus the researchers' attention on different aspects of a student's argument and examine the ways in which technology was used in response to a specific mathematical task.

Methods and Data Sources

For this study, five participants were selected to participate in a series of three interviews conducted by the first two authors of this paper. The third and fourth authors were involved in the teaching of the college geometry course and the development of the interview tasks and protocols.. The five participants included two students pursuing bachelors degrees in mathematics: Teresa and Calvin, and three students pursuing degrees in mathematics education: Gail, Ed, and Will. The one-hour interviews were conducted outside of class during the beginning, middle and near the end of the semester during which time they were taking the college geometry course which included five technology-based lab assignments. A video-camera captured students' written work and a video-recording device directly captured students' work on the computer. Interview transcripts were created from the videotapes. Data taken from the second of these three interviews were analyzed for this study

To analyze college students' mathematical arguments, Toulmin's model of argumentation was used to diagram claims a student made, isolate data the student used to support their claims, identify warrants the student provided to explain how the data were related to the claims and describe the backing if such backing was provided by the student. For each claim that was made by the student, the way in which the technology was used in various aspects of the argument were noted and the type of task on which the student was working was determined.

Because an argument that a student may provide in an interview setting is likely influenced by the task posed, once arguments were identified, the task on which students were working was also coded using the Mathematical Task Coding Instrument (MATCI, Heid, Blume, Hollebrands & Piez, 2002; Hollebrands & Heid, 2005). Tasks that were identified were of three different types: 1) the original task in the interview protocol, 2) questions posed by the interviewer, and 3) tasks taken on by the student that may have deviated from the original task presented to the student or a question posed by the interviewer. Task codes that were used included: Identify, Describe, Elaborate, Produce, Corroborate, Predict, Justify, Generalize, and Generate. These task codes are associated with three different categories: Concept, Product and Reasoning. Codes within the Concept category included Identify, Describe, Elaborate, each of these task has a goal of characterizing a mathematical concept. Codes within the Product category included Produce, Generate, Predict, and Generalize. All four of these tasks require students to create a specific mathematical object. The third category, Reasoning included the Justify and Corroborate tasks and these tasks involve students in developing a rationale for a particular claim. Task codes were placed on each Toulmin diagram to identify the task on which students were working as they were constructing an argument. A spreadsheet was created for each student to coordinate the different codes and this enabled the researchers to look for patterns in the codes.

Results

In the process of analyzing mathematical arguments, themes related to students' uses of technology and their relationships to different components of the argument and the tasks on which they worked were identified. One theme that became evident is that when students used the dragging feature of the software, the students were involved in tasks that were coded as Produce and Justify or Generalize. A second theme that was identified was students' uses of the appearances of diagrams on the screen as warrants and measures used as backing in conjunction with their work on a sequence of tasks coded as Predict, Produce, Describe. A third theme that was noted was that when students were involved in tasks coded as Justify, the students rarely made use of the technology in their warrants and backing. In conjunction with this theme, when measurements were used as backing in the construction of an argument, the tasks in which the students were involved were rarely within the Reasoning category. Elaboration and examples of these three themes will be provided in the following paragraphs.

Dragging in Response to a Produce Task Leading to a Justify or Generalize Task

In the process of constructing arguments, students made use of the dragging feature. This feature not only allowed students to view the change in the appearance of the diagram, but also allowed students to focus their attention on the ways in which the measures that were previously taken changed or remained invariant as they dragged. The dragging of a point in the diagram and the links between the diagram and measurements became data on which students based their arguments. In general, students were engaged in two types of tasks while employing the dragging feature. The first type of task was a production task students responded to by producing variations of a particular geometrical figure or by producing a completely different geometrical figure. During these production tasks, or upon its completion, students would then begin to make generalizations or justifications based on what they noticed.

To illustrate this theme, an example of how one student employed the drag feature to produce data for an argument and how this led to a generalization is provided. During the interview, Teresa was exploring the properties of a Lambert quadrilateral, a quadrilateral with exactly three right angles (see Figure 1). She was trying to determine whether there was a relationship between the lengths of the opposite sides of a Lambert quadrilateral. She noticed that a base of the quadrilateral ($\overline{AB}$), defined as a side contained between the two right angles, and its opposite side ($\overline{EG}$) were both longer than the other two sides ($\overline{AG}$ and $\overline{EB}$). Teresa dragged point A towards point B resulting in a Lambert quadrilateral such that $\overline{EB}$ and $\overline{AG}$ were longer than $\overline{AB}$ and $\overline{EG}$.

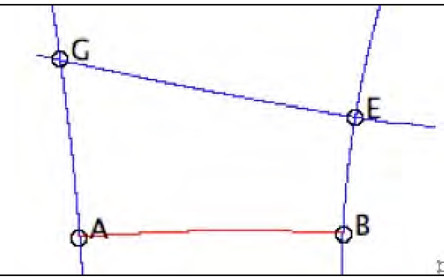

Figure 1. A Lambert quadrilateral, similar to the one used by Teresa and Ed

Teresa then claimed that $\overline{AB}$ and $\overline{EG}$ may not always be longer than $\overline{EB}$ and $\overline{AG}$ (See Figure 2a). In the process of constructing this argument, Teresa was engaged in two types of tasks. The first being a Produce task because she constructed a new example of a Lambert quadrilateral. The second task was a Generalize task because she stated that for all Lambert quadrilaterals that the base of the quadrilateral and its opposite side may not always be longer than the other pair of opposite sides.

Data

*Drag to create a different Lambert quadrilateral**

Claim

"AB and EG may not always be longer than EB and GA"

Measures and/or the appearance of the figures

Warrant/Backing

**italics denotes the use of technology*

Figure 2a: Teresa's argument

Data

Drag a point and view the appearance of the figure and the linked measurements

Claim

"I think it's always going to be acute"

Warrant: "*The measurement is always less than 90*"

Backing: "*As long as it is on that perpendicular it's a defined angle whenever it's not its all of the angles except for the first right angle becomes undefined*"

Warrant/Backing

Figure 2b: Ed's argument

Similar to Teresa's use of the drag feature to produce data for an argument, Ed employed this feature as well to construct an argument regarding the measure of the non-right angle of a Lambert quadrilateral. He dragged point *E* towards point *B* and noticed that measure of angle *G* remained acute. The data for this argument are the appearance and measurements of the changing Lambert quadrilateral afforded by the drag feature (See Figure 2b). He claimed that angle *G* will always be acute and he based this on the fact that the angle measure was always less than 90 degrees, which he used as his warrant. He backed up this warrant by demonstrating that when the point *G* was not on the perpendicular (it was actually on point *A)* only one right angle remained on the screen and the Lambert quadrilateral was destroyed. Similar to Teresa, Ed was involved in two types of tasks, the first being a Produce task, the second a Justify task.

Appearance as Warrants and Measures as Backing

In the process of constructing arguments, there were many instances when students used the diagram on the screen as data, the appearance of the diagram as a warrant and the measurements as backing. During these types of arguments the students appeared to be engaged in three distinct tasks. First, the students Predicted what they believe is true based on the appearance of the diagram. The students would then Produce diagrams and/or measurements that would either confirm or discount their predictions. Lastly, the students would Describe the relationship between their measurements and their prediction.

The following examples illustrate how students used appearances as warrants and measures as backing while engaged in a Predict, Produce, Describe task sequence. During the interview Will was asked to determine whether the diagonals of a parallelogram in the Poincaré disk model always bisect each other. In a previous task, he had constructed a parallelogram and he used this diagram to Predict that the diagonals probably do not bisect each other based on the appearance of the figure on the screen. He then used the Constructions menu to Produce the diagonals of the parallelogram, and the point where the diagonals intersect. He measured the diagonals and stated that the diagonals do not bisect each other because the measures are not equal (See Figure 3a).

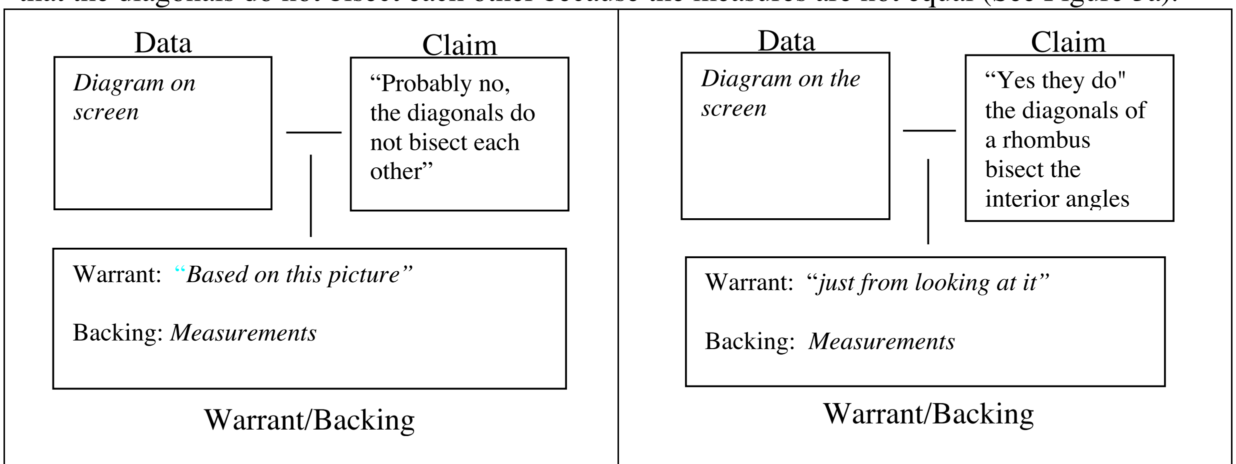

Figure 3a: Will’s argument	Figure 3b: Calvin’s argument

In this example, the appearance of the diagram created using technology was used as a warrant that was backed up with measures while the student was responding to a sequence of tasks that involved predicting, producing, and describing.

As a second example, Calvin was determining whether the diagonals bisect the interior angles of a rhombus in the Poincaré disk model. In a previous task, Calvin had constructed a rhombus and its diagonals. With a focus on the appearance of the diagonals and rhombus, Calvin first predicted that the diagonals would bisect the interior angles. He stated that to make sure he was correct, he would produce some additional measurements. He found the measurements of these angles of interest and described their relationship by stating that the measurements were the same. Similar to Will, Calvin used the technology to produce a diagram and measurements. The appearance was used as a warrant, measurements were used as backing, and this took place in the sequence of tasks that involved Predicting, Producing and Describing.

The Absence of Technology Use as Part of the Warrant and Backing for Justification tasks

In the process of constructing arguments in which the students were engaged in Justification tasks, in general, the warrants and backing of their arguments did not involve the use of technology. However, at times, the students did make use of the constructed figure on the screen and previous measurements as their data. When the students did make use of the technology in their backing, specifically measurements, the students were involved in tasks that fall under the Product and Concept categories of the MATCI task framework. There was only one instance when a student used measures as backing when the student was engaged in a Reasoning task.

During the interview, Gail was provided a task to determine and justify whether a Saccheri quadrilateral, a quadrilateral with exactly two consecutive right angles and a pair of congruent opposite sides that share a ray of a right angle (See Figure 4), could also be a Lambert quadrilateral in Hyperbolic space.

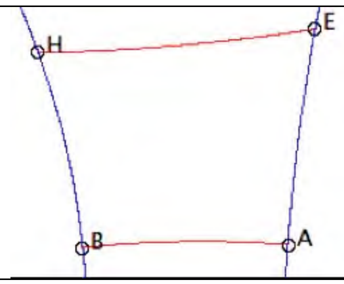

Figure 4. An example of a Saccheri quadrilateral.

In a previous task, she had constructed a Lambert quadrilateral on the computer screen and its appearance was used along with the definitions and properties of these two quadrilaterals, as data for her claim that a Saccheri quadrilateral cannot be a Lambert quadrilateral. She based this argument on the fact that if she moved the Lambert quadrilateral so that it had congruent sides, then the summit angles would both be 90, which would result in a quadrilateral with four 90 degree angles. She backed up her warrant using a property of quadrilaterals that sum of the angles of a quadrilateral in Hyperbolic Geometry must be less than 360 degrees, which she had learned previously in class (See Figure 5a). Gail imagined what would happen if opposite sides of a Lambert quadrilateral were congruent and used a property learned previously as backing.

Figure 5a	Figure 5b
Data: *Diagram on screen* Definitions of Saccheri and Lambert	**Data:** *Saccheri quadrilateral on the screen* and the properties and definition
Claim: "You can't do it"	**Claim:** "Then you have a quadrilateral with 4 congruent angles"
Warrant/Backing: Warrant: "if the sides are congruent and that angle is 90 (angle D) then that angle would have to be 90 (angle F)" Backing: "Sum of the angles of a quadrilateral in Hyperbolic is less than 360"	**Warrant/Backing:** Warrant: "If you put two [Saccheri quadrilaterals] back-to-back." Backing: Since the Saccheri quadrilaterals are congruent the summit angles are congruent
Figure 5a: Gail's argument	Figure 5b: Teresa's argument

In response to a task about whether an equiangular quadrilateral exists in Hyperbolic space, Teresa claimed that it was possible to construct. The data for her argument was a diagram of a Saccheri quadrilateral and its definition and associated properties, which she had noticed in a previous task. Her claim was based on the construction of a second congruent Saccheri quadrilateral that was produced by reflecting the original quadrilateral about its base, the side between the two right angles (See Figure 5b). Teresa had generated a procedure for constructing an equiangular quadrilateral using reflections. She justified her procedure by explaining that the summit angles of congruent Saccheri quadrilaterals will be congruent. In this example, Teresa uses the diagram on the computer screen as data for making a claim about a procedure that could be used to construct an equiangular quadrilateral.

Discussion

Several researchers have described ways in which students view technological evidence as proof (e.g., Chazan, 1993). None of the students in this study held that belief. However, when asked to solve problems that required arguments, students did use the technology in a variety of ways to create evidence that served as data for claims. Often students would be posed with a task from the Reasoning category and they would create subtasks within the Product or Concept category and respond to them by creating arguments where the technology was used as part of a warrant and/or backing. These subtasks were then used to return to the original Reasoning task, where the technology was used in the generation of data and definitions and properties were used as warrants and/or backings. This "breaking down" and "building up" of tasks may have been facilitated by the technology that makes it easy to quickly generate diagrams, and measures linked to those diagrams. By dragging students can generate data from which they can observe what changes and what remains the same, that often led to Generalizations and Justifications. However, when dragging for the purpose of generalizing or justifying, students need to know what to attend to and what to ignore and how to relate what they are observing to what they already know about the particular figure with which they are working.

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