Corrections to

Kogan, I.A., and Olver, P.J., Invariant Euler-Lagrange equations and the invariant variational bicomplex, *Acta Appl. Math.* **76** (2003), 137–193.

Kogan, I.A., and Olver, P.J., The invariant variational bicomplex, *Contemp. Math.* **285** (2001), 131–144.

In the published paper, a sign error in equation (9.43) propagated, affecting the subsequent displayed equation, equations (9.45), (9.46), and particularly (9.47). The same errors appear on p. 142 of the *Contemp. Math.* note.

The corrected version of the affected text follows:

On the other hand,

$$\begin{aligned} d_{\mathcal{V}}\,\varpi^1 &= -\,\kappa^1\,\vartheta\wedge\varpi^1 + \frac{1}{\kappa^1-\kappa^2}\big(\,\mathcal{D}_1\mathcal{D}_2 - Z_2\mathcal{D}_1\,\big)\vartheta\wedge\varpi^2, \\ d_{\mathcal{V}}\,\varpi^2 &= \frac{1}{\kappa^2-\kappa^1}\big(\,\mathcal{D}_2\mathcal{D}_1 - Z_1\mathcal{D}_2\,\big)\vartheta\wedge\varpi^1 - \kappa^2\,\vartheta\wedge\varpi^2, \end{aligned} \tag{9.43}$$

which yields the Hamiltonian operator complex

$$\begin{aligned} \mathcal{B}^1_1 &= -\,\kappa^1, \\ \mathcal{B}^2_2 &= -\,\kappa^2, \end{aligned} \qquad \mathcal{B}^1_2 = \frac{1}{\kappa^1-\kappa^2}\big(\,\mathcal{D}_1\mathcal{D}_2 - Z_2\mathcal{D}_1\,\big) = \frac{1}{\kappa^1-\kappa^2}\big(\,\mathcal{D}_2\mathcal{D}_1 - Z_1\mathcal{D}_2\,\big) = -\,\mathcal{B}^2_1,$$

the equality following from the commutation formula (9.35). Therefore, according to our fundamental formula (9.24), the Euler-Lagrange equations for a Euclidean-invariant variational problem (9.40) are

$$\begin{aligned} 0 = \mathbf{E}(L) = {}& \big[\,(\mathcal{D}_1+Z_1)^2 - (\mathcal{D}_2+Z_2)\cdot Z_2 + (\kappa^1)^2\,\big]\,\mathcal{E}_1(\widetilde{L}) \\ & + \big[\,(\mathcal{D}_2+Z_2)^2 - (\mathcal{D}_1+Z_1)\cdot Z_1 + (\kappa^2)^2\,\big]\,\mathcal{E}_2(\widetilde{L}) + \kappa^1\,\mathcal{H}^1_1(\widetilde{L}) + \kappa^2\,\mathcal{H}^2_2(\widetilde{L}) \\ & + \big[\,(\mathcal{D}_2+Z_2)(\mathcal{D}_1+Z_1) + (\mathcal{D}_1+Z_1)\cdot Z_2\,\big]\cdot\left(\frac{\mathcal{H}^1_2(\widetilde{L}) - \mathcal{H}^2_1(\widetilde{L})}{\kappa^1-\kappa^2}\right). \end{aligned} \tag{9.44}$$

As before, $\mathcal{E}_\alpha(\widetilde{L})$ are the invariant Eulerians with respect to the principal curvatures κ^α, while $\mathcal{H}^i_j(\widetilde{L})$ are the invariant Hamiltonians based on (9.41).

In particular, if $\widetilde{L}(\kappa^1,\kappa^2)$ does not depend on any differentiated invariants, (9.44) reduces to

$$\mathbf{E}(L) = \big[\,(\mathcal{D}_1^\dagger)^2 + \mathcal{D}_2^\dagger\cdot Z_2 + (\kappa^1)^2\,\big]\,\frac{\partial\widetilde{L}}{\partial\kappa^1} + \big[\,(\mathcal{D}_2^\dagger)^2 + \mathcal{D}_1^\dagger\cdot Z_1 + (\kappa^2)^2\,\big]\,\frac{\partial\widetilde{L}}{\partial\kappa^2} - (\kappa^1+\kappa^2)\widetilde{L}. \tag{9.45}$$

For example, the problem of minimizing surface area has invariant Lagrangian $\widetilde{L} = 1$, and so (9.45) gives the Euler-Lagrange equation

$$\mathbf{E}(L) = -(\kappa^1 + \kappa^2) = -2H = 0, \tag{9.46}$$

and so we conclude that minimal surfaces have vanishing mean curvature. As noted above, the Gauss–Bonnet Lagrangian $\widetilde{L} = K = \kappa^1 \kappa^2$ is an invariant divergence, and hence its the Euler-Lagrange equation is identically zero. The mean curvature Lagrangian $\widetilde{L} = H = \frac{1}{2}(\kappa^1 + \kappa^2)$ has Euler-Lagrange equation

$$\tfrac{1}{2}\left[\,(\kappa^1)^2 + (\kappa^2)^2 - (\kappa^1 + \kappa^2)^2\,\right] = -\,\kappa^1\,\kappa^2 = -K = 0. \tag{9.47}$$

For the Willmore Lagrangian $\widetilde{L} = \frac{1}{2}(\kappa^1)^2 + \frac{1}{2}(\kappa^2)^2$, [**3**, **6**], formula (9.44) immediately gives the known Euler-Lagrange equation

$$\mathbf{E}(L) = \Delta(\kappa^1 + \kappa^2) + \tfrac{1}{2}(\kappa^1 + \kappa^2)(\kappa^1 - \kappa^2)^2 = 2\,\Delta H + 4(H^2 - K)H = 0, \tag{9.48}$$

where

$$\Delta = (\mathcal{D}_1 + Z_1)\mathcal{D}_1 + (\mathcal{D}_2 + Z_2)\mathcal{D}_2 = -\,\mathcal{D}_1^\dagger \cdot \mathcal{D}_1 - \mathcal{D}_2^\dagger \cdot \mathcal{D}_2 \tag{9.49}$$

is the Laplace–Beltrami operator on our surface.